Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Transform Theory

Subject Code : 4SC05TTE1 **Branch: B.Sc. (Mathematics)**

Semester : 5 **Date : 02/05/2016** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: periodic function. What is the fundamental period of $\cos nx$? **(02)**

b) Evaluate: $\mathcal{L}[3^t]$. **(02)**

c) Define: Fourier transform. **(02)**

d) Define: Z −transform. **(02)**

e) Check whether the function $f(x) = \sin x^2$ is even or odd function? **(02)**

f) Evaluate: $\mathcal{L}^{-1}\left[\frac{1}{s(s-1)}\right]$. **(02)**

g) State convolution theorem **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the Fourier series of the function **(07)**

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ k, & -1 < x < 1 \\ 0, & 1 < x < 2. \end{cases}$$

b) If $f(t)$ has the Laplace transform $F(s)$, then show that $e^{at}f(t)$ has the transform $F(s-a)$. Hence evaluate $L[e^{-t}(3\cos 20t - 7\sin 20t)]$. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Show that **(07)**

$$\int_0^\infty \frac{\cos xw + w\sin xw}{1+w^2}\,dx = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0. \end{cases}$$

b) Using Laplace transform solve $y'' + 4y' + 5y = 50t, y(0) = 5, y'(0) = -5.$ **(07)**

Q-4 **Attempt all questions** **(14)**

a) Obtain the half range Fourier cosine series of **(07)**

$$f(x)=\begin{cases}\frac{2k}{L}x, & 0<x<\frac{L}{2}\\ \frac{2k}{L}(L-x), & \frac{L}{2}<x<L.\end{cases}$$

b) If $Z(u_n)=U(z)$, then prove that $\lim_{n\to\infty}(u_n)=\lim_{z\to 1}(z-1)U(z)$. **(07)**

Q-5 **Attempt all questions** **(14)**

a) Find the Fourier sine transform of $e^{-|x|}$. Hence show that **(07)**

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2}, m>0.$$

b) Evaluate $\mathcal{L}^{-1}\left[\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right]$. **(07)**

Q-6 **Attempt all questions** **(14)**

a) Express $f(x)=\begin{cases}1, 0\le x\le\pi\\ 0, \quad x>\pi\end{cases}$ as a Fourier sine integral and evaluate **(07)**

$\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda}\sin(x\lambda)d\lambda$.

b) Find the Z −transform of $(i)n\sin n\theta \quad (ii)\, n^2e^{n\theta}$. **(07)**

Q-7 **Attempt all questions** **(14)**

a) If $U(z)=\frac{2z^2+5z+14}{(z-1)^4}$, evaluate u_2 and u_3. **(07)**

b) Obtain Fourier series for the function $f(x)$ given by **(07)**

$$f(x)=\begin{cases}1+\frac{2x}{\pi}, -\pi\le x\le 0,\\ 1-\frac{2x}{\pi}, \quad 0\le x\le\pi.\end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$.

Q-8 **Attempt all questions** **(14)**

a) Find the inverse Laplace transform of $(i)\log\frac{s^2+1}{s(s+1)}$ $(ii)\ \cot^{-1}\left(\frac{s}{2}\right)$. **(07)**

b) Find the Fourier transform of **(07)**

$$f(x)=\begin{cases}1, |x|<1\\ 0, |x|>1\end{cases}.$$

Evaluate $\int_0^\infty \frac{\sin x}{x}dx$.